Example - 13

Antenna supplied with a power of 10 W - calculate the radiated Power & lost power when the efficiency of antenna is 90%.

$P_{in} = 10\ W$

$e_{cd} = 90\%$

$P_{rad} = ?$

$P_{loss} = ?$

<u>Solution</u>:

$\because e_{cd} = \frac{P_{rad}}{P_{in}} \Rightarrow$

$P_{rad} = e_{cd} P_{in} = 10 * 0.9 = 9\ W$

$P_{loss} = P_{in} - P_{rad} = 10 - 9 = 1\ W$

<u>or</u> $100\% - 90\% = 10\% \Rightarrow$

$10 * \frac{10}{100} = 1\ W$

Eample -

Antenna with 80 Ω radiation resistance & 10 Ω loss resistance. Find antenna radiation efficiency & Power losses.

$\because e_{cd} = \frac{R_r}{R_r + R_L} = \frac{80}{80+10}$

$= \frac{80}{90} = 0.888 \Rightarrow$

$R_r = 80\ \Omega$

$R_L = 10\ \Omega$

$e_{cd} = 88.88\% \simeq 89\%$

$P_{loss} = P_{in} + e_{cd} = 100\% - 89\% = 11\%$

Example - #4

An antenna has a field Pattern given by $E(\theta) = \cos^2\theta$ for $0^\circ \leq \theta \leq 90^\circ$. Find the beam area.

$E(\theta) = \cos^2\theta$

$0 \leq \theta \leq \pi/2$

$\Omega_A = ?$

Solution:

$$\Omega_A = \int_0^{2\pi}\int_0^{\pi} P_n(\theta,\phi)\, d\Omega$$

$$P_n(\theta) = |E(\theta)|^2 = |\cos^2\theta|^2 = \cos^4(\theta)$$

$$\Rightarrow \Omega_A = \int_0^{2\pi}\int_0^{\pi/2} \cos^4\theta \sin\theta\, d\theta\, d\phi$$

$$\Rightarrow \Omega_A = 2\pi\left[\frac{\cos^5\theta}{5}\right]_0^{\pi/2} = \frac{2\pi}{5} = 1.26 \text{ sr}$$

Quiz



For a sphere of radius 1 m, find the solid angle (in square radians) of a spherical cap on the surface of the sphere when $0 \leq \theta \leq 30^\circ$, $0 \leq \phi < 360^\circ$

Then fined radiation intensity at 20 watt/$m^2$

Solution

$$* \ \Omega = \int_0^{360}\int_0^{30} d\Omega \equiv \int_0^{2\pi}\int_0^{\pi/6} \sin\theta \, d\theta \, d\phi$$

$$= \int_0^{2\pi} d\phi \int_0^{\pi/6} \sin\theta \, d\theta = 2\pi\left[-\cos\theta\right]_0^{\pi/6} = 0.835 \ sr$$

$$* \ U = W r^2 \equiv 20 * (1)^2 = 20 \ watt/sr$$

Quiz

The radiation intensity of a unidirectional antenna is

$$U = U_m \cos\theta \qquad 0 \leq \theta \leq \pi/2 \; , \; 0 \leq \phi \leq 2\pi$$

Find ① Eaact directivity

② APProximate directivity

solution

① Eact directivity : $D_0 = \frac{4\pi U_{max}}{P_{rad}}$

$\because U = U_m \cos\theta \Rightarrow \therefore U_{max} = U_m$

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi/2} U \, d\Omega = \int_0^{2\pi}\int_0^{\pi/2} U_m \cos\theta \sin\theta \, d\theta \, d\phi$$

$\because \sin 2\theta = 2\cos\theta \sin\theta$

$\therefore \cos\theta \sin\theta$ will be $\frac{1}{2}\sin 2\theta$

$$\therefore P_{rad} = U_m \left[\phi\right]_0^{2\pi} \int_0^{\pi/2} \frac{1}{2}\sin 2\theta \, d\theta \Rightarrow$$

$$P_{rad} = 2\pi U_m * \frac{1}{2}\left[\frac{-\cos 2\theta}{2}\right]_0^{\pi/2} = \pi U_m$$

$$D_0 = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\cancel{\pi} \cancel{U_m}}{\cancel{\pi} \cancel{U_m}} = \boxed{4}$$

(5)

(2) Approximate directivity : $D_o \simeq \dfrac{41253^{\square}}{\theta_E \, \theta_H}$

$\because$ HPBW is a point where power becomes half, so

$\cos\theta = 0.5 \Rightarrow \theta = \cos^{-1}(0.5) = 60^\circ$

$HBBW = 2 * 60 = 120^\circ$

$\theta_E = 120^\circ = \theta_H \Rightarrow$

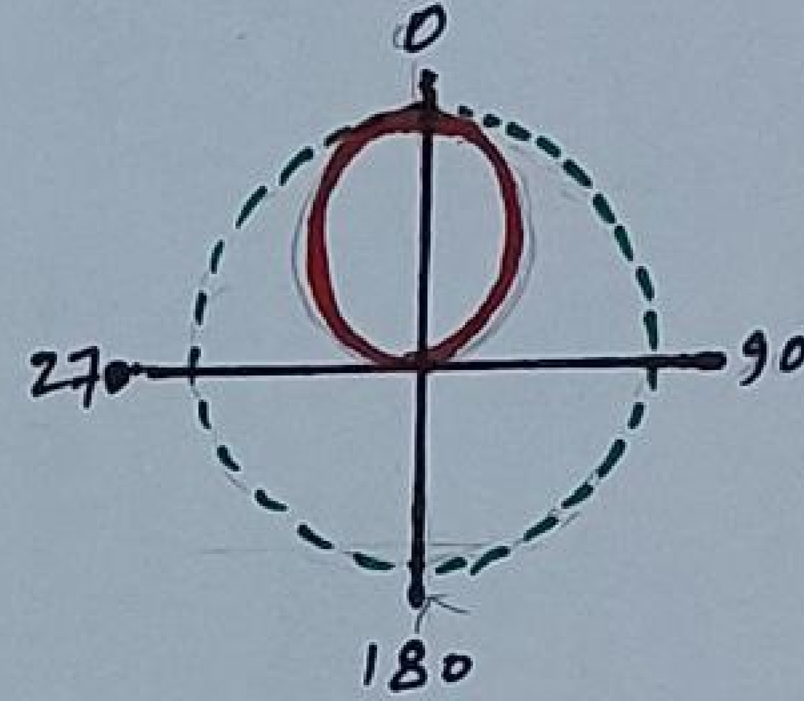
$$D_o \simeq \frac{41253^{\square}}{\theta_E \, \theta_H} = \frac{41253^{\square}}{120^\circ * 120^\circ} = \boxed{2.865}$$

Eample - 15

For unidirectional cosine Pattern, calculate D.

Solution

∵ we have unidirectional Pattern antenna, that is, $\theta$ is determined from 0 to $\pi/2$.




also, unidirectional Pattern means, the Pattern represents radiation intensity with maximum value in cosine direction $\Rightarrow U = U_m \cos\theta$

$$D_o = \frac{4\pi U_m}{P_{rad}}$$

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi/2} U \sin\theta \, d\theta \, d\phi = \int_0^{2\pi}\int_0^{\pi/2} U_m \cos\theta \sin\theta \, d\theta \, d\phi$$

$$= U_m \int_0^{2\pi} d\phi \int_0^{\pi/2} \cos\theta \sin\theta \, d\cancel{\phi}$$

∵ $\sin 2\theta = 2\cos\theta \sin\theta \Rightarrow \cos\theta \sin\theta$ in eq. above becomes $\frac{\sin 2\theta}{2}$, $\int \sin 2\theta = -\frac{\cos 2\theta}{2} \Rightarrow$

$$P_{rad} = U_m \int_0^{2\pi} d\phi \int_0^{\pi/2} \frac{\sin 2\theta}{2} \, d\theta = \frac{U_m}{2}\left[\phi \Big|_0^{2\pi}\right]\left[-\frac{\cos 2\theta}{2}\Big|_0^{\pi/2}\right] \Rightarrow$$

$$P_{rad} = \frac{U_m}{4}\left[2\pi\right]\left[-\cos 2 * \frac{\pi}{2} + \cos 2 * 0\right] \Rightarrow$$

$$P_{rad} = \frac{U_m}{4}\left[2\pi\right]\left[-(-1)+1\right] = \frac{\cancel{4}\pi U_m}{\cancel{4}} = \underline{\underline{\pi U_m}}$$

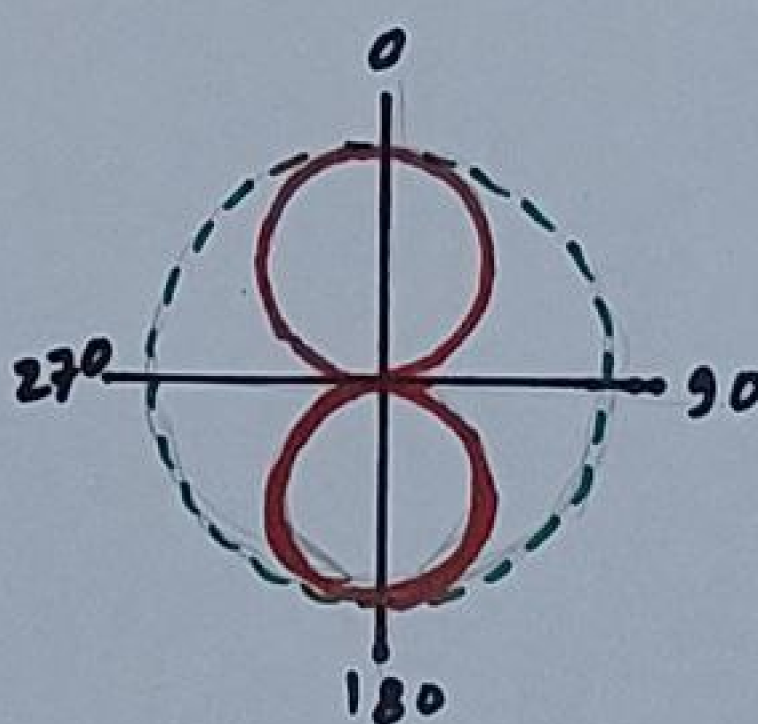
$$\therefore \quad D = \frac{4\pi U_m}{P_{rad}} = \frac{4\cancel{\pi U_m}}{\cancel{\pi U_m}} = \underline{\underline{4}}$$

Example - 16

For Bidirectional cosine Pattern antenna, calculate directivity.

Solution

$\because$ we have bidirectional Pattern antenna, that is, $\theta$ is determined from 0 to $\pi$, & the Pattern represents radiation intensity with maximum value in cosine direction $\Rightarrow U = U_m \cos\theta$




$$D_o = \frac{4\pi U_m}{P_{rad}}$$

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} U \sin\theta \, d\theta \, d\phi = \int_0^{2\pi}\int_0^{\pi} U_m \cos\theta \sin\theta \, d\theta \, d\phi$$

$$= U_m \int_0^{2\pi} d\phi \int_0^{\pi} \cos\theta \sin\theta \, d\theta$$

$\because$ $\sin 2\theta = 2\cos\theta\sin\theta \Rightarrow \cos\theta\sin\theta$ in eq. before becomes $\frac{\sin 2\theta}{2}$

$$\Rightarrow \boxed{P_{rad} = U_m \int_0^{2\pi} d\phi \int_0^{\pi} \frac{\sin 2\theta}{2} d\theta} = U_m [\phi]_0^{2\pi} \left[\frac{-\cos 2\theta}{2}\right]_0^{\pi}$$

$$\Rightarrow P_{rad} = \frac{U_m}{2}(2\pi)[-\cos 2 * \pi + \cos 0] \Rightarrow$$

$$P_{rad} = U_m \pi[-1+1] = \underline{\underline{0}}$$

$$\Rightarrow D = \frac{4\pi U_m}{P_{rad}} = \frac{4\pi U_m}{0} = \underline{\underline{\infty \; !!!}}$$

But directivity can not be $\infty$ in practical cases !!

Therefore, we should change (manipulates) the limites of integration by using property of definete integrals (the same meaning be presesved i.e, physically un changed) $\Rightarrow \int_0^{\pi} x\,dx = 2\int_0^{\pi/2} x\,dx$ where $x$-odd func-

so, the eq. before,

$$P_{rad} = U_m \int_0^{2\pi} d\phi \int_0^{\pi} \frac{\sin 2\theta}{2} d\theta \xrightarrow{\text{becomes}} U_m \int_0^{2\pi} d\phi \; 2\int_0^{\pi/2} \frac{\sin 2\theta}{2} d\theta$$

$$\Rightarrow P_{rad} = U_m [\phi]_0^{2\pi} \left[\frac{-\cos 2\theta}{2}\right]_0^{\pi/2} = \frac{U_m}{\cancel{2}}(\cancel{2}\pi)\left[-\cos \cancel{2} * \frac{\pi}{\cancel{2}} + \cos 2 * 0\right]$$

$$\Rightarrow P_{rad} = U_m[-1(-1)+1] = \underline{\underline{2\pi U_m}} \Rightarrow D = \frac{4\pi U_m}{P_{rad}} = \frac{4\pi U_m}{2\pi U_m} = \underline{\underline{2}}$$

* It is half "D" of unidirectional antenna.